Material implication is problematic if I say it is

Henning Strandin

2015-11-03

These are just some thoughts on how we teach logic to beginners. My experience is from introductory logic courses provided to philosophy students, I'm sure it's done differently in other contexts.

The claim in the title is false. My saying there's something wrong with material implication doesn't make it so. But if the claim is understood to have the form of a material implication, then it is true if material implication is problematic—independently of what I say and why.

This aspect of material implication, that its truth conditions are such as to ignore any connection between the antecedent and the consequent, has been regarded as a problem by some people since the dawn of modern symbolic logic around the turn of the last century. (How to understand the truth conditions for a conditional statement has been debated throughout the history of logic theories.)

A quick recap: In truth-functional logic, material implication is indicated by the symbol '$\rightarrow$' (or sometimes by '$\supset$' or '$\Rightarrow$', but I will use the single arrow), and 'p $\rightarrow$ q' is read as 'if p then q.' The expression is true if and only if it is not the case that p is true and q is false. Hence

If Russia won the race to the moon then The Avengers 3 premiers tonight

would be true if interpreted as a material implication (since the antecedent is false), and

If Donald Trump becomes president, then peace will reign on Earth

is true, again if understood as a material implication, if peace will reign on Earth—even if Trump has nothing to do with it. But in their ordinary, non-technical, senses these statements are not true under such conditions.

For the longest time I used to think there was nothing, and could not *be* anything, wrong with material implication. After all, it has well-defined truth conditions, that are functions of the truth values of the sentences connected by the arrow, and what more could you ask of a truth functional connective?

Table 1: Truth conditions for material implication

p	q	p → q
True	True	True
True	False	**False**
False	True	True
False	False	True

But one day I realized that I had changed my mind. I was having some serious issues with material implication. The reason was one, and one only: teaching introductory logic courses.

In introductory propositional logic, we teach the five logic connectives: negation ('~'), conjunction ('∧'), disjunction ('∨'), implication ('→'), and equivalence ('↔'). It is usually necessary to repeat a few times to students that disjunction is 'inclusive.' That is to say, that 'p ∨ q' is true if both p and q are true—this is in contradiction with some everyday uses of 'or.' It may trip some students up a couple of times at first, but it's not a great hurdle. Negation, conjunction, and equivalence are usually not a problem.

However, ask the students halfway through the course if p → ~p is a contradiction, and around half, sometimes more, will say yes. Because 'if p then not p' really does look like a contradiction, even when you've had the truth conditions for material implication repeated to you several times. (It is not a contradiction, of course, but equivalent to ~p.) For implication only do I have to resort to tables of translations to natural language and Venn diagrams to promote understanding.

My problem with material implication is pedagogical, then. Now, many things are hard to learn in the beginning, and there isn't much you can do about that. And material implication is central in the single historically most important inference rule in logic: *Modus Ponens*.

Modus Ponens is also a very natural inference rule. If it's Thursday, it's pea soup for lunch. It's Thursday. So, it's pea soup for lunch. What could be more obvious? That's part of the problem. You can understand and use Modus Ponens perfectly for every case you come across, and still be using a stronger version of 'if… then…' than material implication, and so still be thrown off by 'p → ~p'.

Table 2: Modus Ponens

Premise 1:	$p \rightarrow q$
Premise 2:	p
Conclusion:	q

So, I have a suggestion. Let's not teach material implication, not at first. Let's only teach negation, conjunction, and disjunction. (You can do everything with these three.) But what about Modus Ponens? Let's teach Disjunctive Syllogism instead!

Table 3: Disjunctive Syllogism

Premise 1:	$p \vee q$
Premise 2:	~p
Conclusion:	q

It may not be as obvious as Modus Ponens, but once you've learned how to use it, there is little risk of misunderstanding. Then, when our beginning students have mastered these symbols and the inference rule, we can explain that $p \rightarrow q$ is equivalent to ~p $\vee$ q, and that both Modus Ponens and its sibling Modus Tollens can be understood as special cases of the Disjunctive Syllogism. The arrow does not stand for 'if… then…' in any ordinary sense—it's a particular disjunction that satisfies only the truth functional part of the ordinary conditional. Or, more straightforwardly: *the conditional is not a form in truth-functional logic.* Give students a chance to grasp this from the very beginning.

From The Blog at henningstrandin.me

Email: henning.strandin@gmail.com