The other solution to the base unit discrepancy

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This text originally appeared on a web forum dedicated to decimal time keeping systems, sometime in 2004. To my delight several of the users of that forum took my proposal seriously.

1 The Base unit mismatch

It seems generally accepted that because life on Earth tends to repeat in both daily and annual cycles, these two both need to be counted within a useful time keeping system. In effect, the system must be constructed so that there are two units, *day* and *year*, that correspond to the two astronomical cycles of Earth axis rotations and Earth solar revolutions, in such a way that *day* is a subunit of *year*.

A decimal system is by definition a system where every unit is an exponent of 10. This means that every unit has a ratio to all of its sub- and superunits that is also an exponent of 10. (E.g., 1 kilometer / 1 meter = 10^3, and 1 kilogram / 1 hectogram = 10^1.)

Cyclic earth time dictates two units to us, that do not have a ratio that is compatible with a decimal system. There are approximately 365.25 days in a year, and that is not a power of 10. This is a fixed premise of the problem, and can't be changed.

2 The reason for a decimal system

We prefer to use decimal systems for measurement because our number system is itself decimal. Powers of 10 are special in our decimal, positional number system in that they can be generated by simply inserting zeroes and/or moving the decimal point. If we use only decimal units for measurements as well, we use the same trivial procedure to translate between units and their sub- or superunits. With non-decimal units, we need to make calculations for each translation, or look them up in a table.

For mass, volume, and temperature, we are free to define our own units and subunits, and there is no reason to not make them decimal. As far as I know, time is the only case of measurement where we are given two units with a rational relationship by nature, as part of the premise. Because of this, efforts to make time keeping systems conform to our decimal number system and other decimal units will forever be imperfect.

3 The other solution

However, the very fact that we are free to define *all* other units as we please is the clue to a solution. If it is indeed the case that only in time measurements are we bound by a ratio in nature, then it would seem to make more sense to adapt our other unit systems to this ratio, than to accept a flawed and incompatible time keeping system. It's within our abilities to change the base of our number system and of all of our other units, while the number of days in a year is fixed.

4 A new base

We then need to examine what new numerical base the ratio of days to years suggests to us. First of all, an exponentiation of a whole number is always a whole number, so 365.25 can't be the exponentiation of any whole number. The base of a number system is by definition the number of signs it uses, so it must be a positive integer, a whole number. We can't use 7.3 signs in the number system. It seems obvious then that to have any chance of succeeding, we need to look at 365 rather than 365.25. We'll get back later to the implications of this.

We now notice that the square root of 365 is approximately 19.1. If it had been a whole number, that would have been our candidate for a new numerical base. Since 365 is such a cumbersome number however (can only be factored into 5 and 73, both primes), there seems to be no smaller number that we can use as our base. We are stuck with a new numerical base of 365.

5 Practical issues with having 365 as a numerical base

First of all, we will need 365 distinct signs to use in our new numerical system. This is actually a smaller problem than one might imagine. We can easily derive 365 signs from our existing number system, by combining the signs that we use to build the same numbers today. The signs for the numbers 12, 75, and 103 (for example), would then look something like this:

These signs will all be easy to recognize and remember for someone familiar with our current system. I leave it as an exercise for the reader to come up with a convenient way to draw these signs without lifting the pencil from the paper. (You might want to revisit the Bridges of Konigsberg problem.)

Another potential issue is how to build input devices for our trecentisexagintapentimal number system. Using an average decimal calculator as a model, the size of the keyboard would be about 10x30 centimeters, perhaps larger, since the trecentisexagintapentimal signs are somewhat more complex than their decimal counterparts. This is however also a problem to which a solution easily presents itself. Most Personal Digital Assistants today are equipped with a writing pad, for entering signs into the device using a pen. A similar contraption can be used for calculators to enter numerical signs, instead of having a button for each one.

6 Benefits of using 365 as a numerical base

In these times when oil prices are increasing, computers are becoming faster and faster, average global temperatures are steadily rising, and people are getting fatter at an increasing rate, it is foresighted to switch even now to a numerical system that can express large numbers with fewer signs. In our current system, we need to employ three signs already at 100 units. With the trecentisexagintapentimal system, we can use two signs up to no less than 133,224 (expressed decimally) units! Writing space is an exhaustible resource, and using 365 as our base, we significantly push forward the inevitable date of depletion.

Another benefit of the trecentisexagintapentimal system is that it happens to be compatible with our planet's day/year ratio.

7 Leap units

What about that 0.25 leftover? Right. There are after all not 365 days in a year, but approximately 365.25. With a year of 365 days, we will "lose" one actual day every four years or so. Since we have had to use whole-number units in our calendars (for the reason mentioned above), we have needed to round the number of days per year to the nearest whole number. However, this truncation leads to an unacceptable displacement of the calendar over the years.

Hence, it's not possible to simply use the base 365 and leave it at that, as convenient as that would have been. Our astronomically based number system must compensate for the fact that there is not a whole number of Earth axis rotations in one solar revolution. Fortunately, there is a simple and sufficient algorithm to accomplish this. Append a leap sign, we call it "366," at the end of the regular number series, when one of the following criteria are fulfilled:

Append a leap sign, "366," if and only if

the next upcoming multiple of 365, divided by 365, is exactly divisible by 4, but not exactly divisible by 100, or is exactly 1

or

the next upcoming multiple of 365, divided by 365, is exactly divisible by 100 and exactly divisible by 400.

This algorithm can easily be built into all calculators, thermometers, scales, rulers, and so on, and will finally make them consistent with our time keeping units, which is good since mathematical consistency is the only thing that matters in this world.

8 Final words

The solution to the age-old problem of inconsistent base units between time and other measurements can be conveniently resolved by recognizing that nature only dictates one ratio, that of days to years, and that all other units, and the number system itself, use an arbitrary base. Hence, our universal numerical base should be derived from that ratio, to achieve complete and universal mathematical consistency between all cases of measurement and calculation. The change to a trecentisexagintapentimal system has the additional benefit of conserving writing space, a resource that will inevitably become scarcer as literacy rates increase.

As the formal name of this system, taking into account the included leap unit feature, I suggest: *Trecentisexagintapentimal*+.

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